Spiral plat sans courbes terminales

Déformée élastique en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $ép = 0.03\,mm$ $ha = 0.15\,mm$ $S = 4.5 \times 10^{-3}\,mm^2$ $TOL := 10^{-9}$

$d2_{sp} = 4.52\,mm$ $d1_{sp} = 1.1\,mm$ $p_{sp} = 0.135\,mm$ $n_{sp} = 12.667$

$L := L_{sp}$ $L = 11.182\,cm$ $\psi_0 := 2\cdot\pi\cdot n_{sp}$ $\psi_0 = 4.56 \times 10^3\,deg$

Position du piton $r_P := 0.5\cdot d2_{sp}$ $\alpha_P := 0$ $x_P := r_P\cdot cos(\alpha_P)$ $y_P := r_P\cdot sin(\alpha_P)$

$x_P = 2.26\,mm$ $y_P = 0\,mm$ $z_P := x_P + i\cdot y_P$

Position du point d'attache à la virole $r_V := 0.5\cdot d1_{sp}$ $\alpha_V(\theta) := \psi_0 + \theta$ $x_V(\theta) := r_V\cdot cos(\alpha_V(\theta))$ $y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Forme naturelle du spiral

$a := \frac{p_{sp}}{2\cdot\pi}$ $r_s(\alpha) := r_P - a\cdot\alpha$ $x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha)$ $y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$ $z_0(\alpha) := r_s(\alpha)\cdot exp(i\cdot\alpha)$

$r'(\alpha) := \frac{d}{d\alpha}r_s(\alpha)$ $s(\alpha) := \frac{\pi}{p_{sp}}\cdot\left(r_P^2 - r_s(\alpha)^2\right)$ $s(\alpha) := r_P\cdot\alpha - \frac{a}{2}\cdot\alpha^2$ $s(\psi_0) = 11.182\,cm$

Amplitude stationnaire du balancier $\theta_0 = 270\,deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$ $W_{f3} := W_{f\_rect}(ép, ha)$ $\sigma_{max} := \frac{E\cdot I_{33}}{L\cdot W_{f3}}\cdot\theta_0$ $\sigma_{max} = 132.293\,\frac{N}{mm^2}$

Déformée du spiral avec la virole non liée à l'axe de balancier

$$z_1(\theta,\alpha) := z_P + \int_0^{\alpha} exp(i\cdot\varphi_0(\alpha'))\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha')}{L}\right)\cdot r_s(\alpha)\, d\alpha'$$

$$z'_0(\alpha) := \left[-a + i\cdot(r_P - a\cdot\alpha)\right]\cdot exp(i\cdot\alpha)$$

$$z_1(\theta,\alpha) := z_P + \int_0^{\alpha} z'_0(\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha')}{L}\right) d\alpha'$$

$$z_1(\theta,\alpha) := z_P + \int_0^{\alpha} \left[-a + i\cdot(r_P - a\cdot\alpha')\right]\cdot exp\left[i\cdot\alpha'\cdot\left[1 + \frac{\theta}{L}\cdot\left(r_P - \frac{a}{2}\cdot\alpha'\right)\right]\right] d\alpha'$$

Graphe de la déformation

Forme naturelle

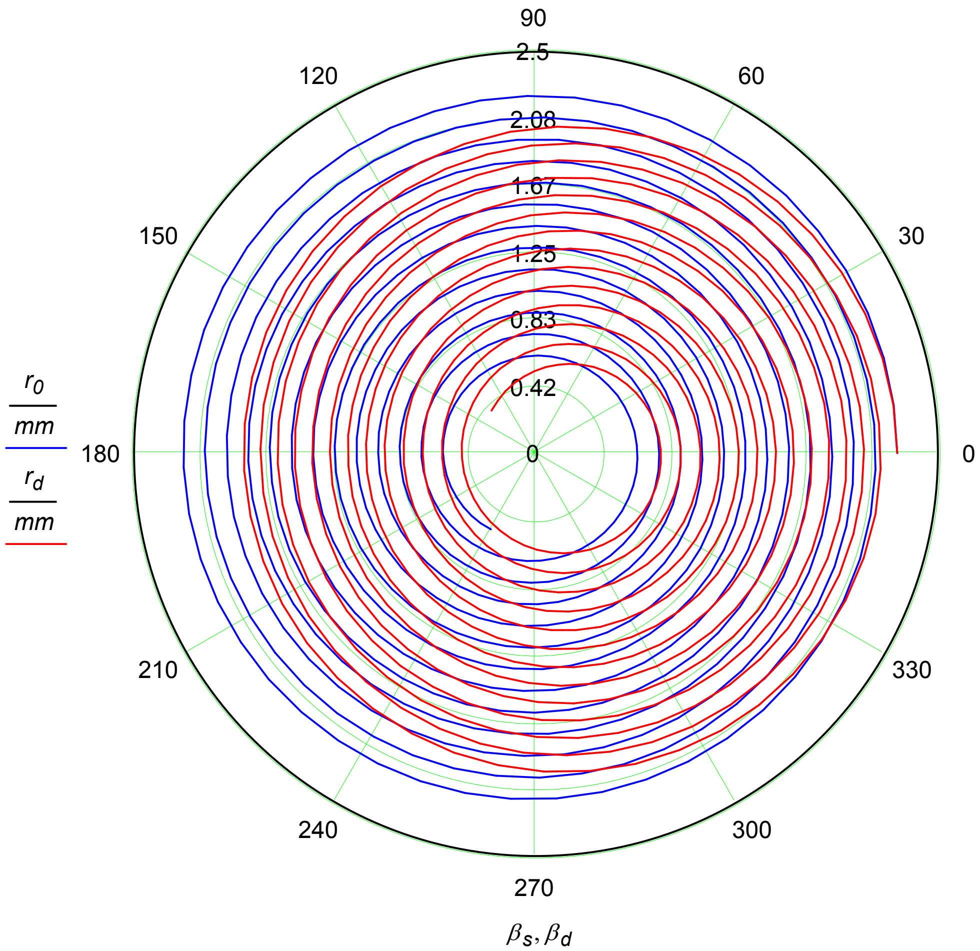
$n := 50 \cdot partentière(n_{sp}) + 1 \quad i := 0 .. n-1 \quad \Delta\alpha := \dfrac{\psi_0}{n-1} \quad \alpha_i := i \cdot \Delta\alpha$

$x_{0_i} := x_{0s}(\alpha_i) \quad y_{0_i} := y_{0s}(\alpha_i) \quad r_0 := \overrightarrow{\sqrt{x_0^{\,2} + y_0^{\,2}}} \quad \beta_s := \overrightarrow{Atan(x_0, y_0)}$

Déformée

$z_{d_i} := z_1(\theta_0, \alpha_i) \quad n_{pt} := dernier(z_d) \quad x_d := Re(z_d) \quad y_d := Im(z_d) \quad r_d := \overrightarrow{|z_d|} \quad r_{d_{n_{pt}}} = 0.378\, mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)} \quad \beta_{d_0} = 0\, deg \quad \beta_{d_{n_{pt}}} = 135.516\, deg \quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 150\, deg$



$x_V(\theta_0) = -0.476\, mm \quad x_{d_{n_{pt}}} - x_V(\theta_0) = 0.207\, mm \quad y_V(\theta_0) = 0.275\, mm \quad y_{d_{n_{pt}}} - y_V(\theta_0) = -0.01\, mm$

Déplacement de la virole libre

$$\mathbf{\Delta_1}(\theta) := \frac{\mathrm{i}\cdot\theta}{L} \cdot \int_0^{\psi_0} exp\left[\mathrm{i}\cdot\alpha\cdot\left[1 + \frac{\theta}{L}\cdot\left(r_P - \frac{a}{2}\cdot\alpha\right)\right]\right]\cdot(r_P - a\cdot\alpha)^2\, d\alpha$$

$u_1(\theta) := Re(\mathbf{\Delta_1}(\theta)) \quad v_1(\theta) := Im(\mathbf{\Delta_1}(\theta)) \quad u_1(\theta_0) = -0.207\, mm \quad v_1(\theta_0) = 0.01\, mm$

Calcul des réactions sur la virole

$$\xi_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} x_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha \qquad \eta_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha$$

$$q2_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} y_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad p2_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha \qquad k_{0s} := \frac{1}{L}\cdot\int_0^{\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha$$

$$\xi_{0s} = -1.361\times 10^{-3}\,mm \qquad \eta_{0s} = 0.047\,mm \qquad q2_{0s} = 1.352\,mm^2 \qquad p2_{0s} = 1.353\,mm^2 \qquad k_{0s} = 0.026\,mm^2$$

$$\boldsymbol{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} - \eta_{0s}^{\,2} & \eta_{0s}\cdot\xi_{0s} - k_{0s} \\ \eta_{0s}\cdot\xi_{0s} - k_{0s} & p2_{0s} - \xi_{0s}^{\,2} \end{pmatrix} \qquad \boldsymbol{R'}(\theta) := \boldsymbol{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad \boldsymbol{R'}(\theta_0) = \begin{pmatrix} -9.673\times 10^{-5} \\ 2.943\times 10^{-6} \end{pmatrix} N$$

$$\left|\boldsymbol{R'}(\theta_0)\right| = 9.678\times 10^{-5}\,N$$

Approximations de Haag

$$\boldsymbol{OP} := r_P \qquad \boldsymbol{OV} := r_V\cdot e^{i\cdot\psi_0} \qquad \boldsymbol{\Delta 1}(\theta_0) = -0.207 + 0.01i\,mm$$

$$\boldsymbol{w}(\theta) := \frac{-\theta}{L}\cdot\left[\left(r_P - 2\cdot i\cdot a - \frac{\theta}{L}\cdot r_P^{\,2}\right)\cdot\boldsymbol{OP} - \left(r_V - 2\cdot i\cdot a - \frac{\theta}{L}\cdot r_V^{\,2}\right)\cdot\boldsymbol{OV}\cdot e^{i\cdot\theta}\right] \qquad \boldsymbol{w}(\theta_0) = -0.205 + 0.011i\,mm$$

$$q2_{0s} = 1.352\,mm^2 \qquad q2_{0s} - \eta_{0s}^{\,2} = 1.35\,mm^2 \qquad p2_{0s} = 1.353\,mm^2 \qquad p2_{0s} - \xi_{0s}^{\,2} = 1.353\,mm^2$$

$$\sigma 2 := \frac{1}{L}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\overline{z_0(\alpha)}\cdot r_s(\alpha)\,d\alpha \qquad \sigma 2 := \frac{1}{2}\cdot\left(r_P^{\,2} + r_V^{\,2}\right) \qquad \sigma 2 = 2.705\,mm^2$$

$$R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma 2}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} -9.658\times 10^{-5} \\ 4.806\times 10^{-6} \end{pmatrix} N \qquad \left|R'(\theta_0)\right| = 9.67\times 10^{-5}\,N$$

$$\boldsymbol{F}(\theta) := \frac{-2\cdot E\cdot I_{33}}{L^2\cdot\sigma 2}\cdot\theta\cdot r_P\cdot\left[\left(1 - \frac{\theta}{L}\cdot r_P\right)\cdot\boldsymbol{OP}\right] \qquad \boldsymbol{F}(\theta_0) = -9.095\times 10^{-5}\,N$$

Deuxième approximation de la déformée du spiral

$$x_1(\theta,\alpha) := Re\left(z_1(\theta,\alpha)\right) \qquad y_1(\theta,\alpha) := Im\left(z_1(\theta,\alpha)\right) \qquad R'_x(\theta) := \boldsymbol{R'}(\theta)_0 \qquad R'_y(\theta) := \boldsymbol{R'}(\theta)_1$$

$$s\xi_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')\cdot r_s(\alpha')\,d\alpha' \qquad \xi_{1s}(\theta) := \frac{1}{L}\cdot s\xi_1(\theta,\psi_0) \qquad \xi_{1s}(\theta_0) = 0.199\,mm$$

$$s\eta_1(\theta,\alpha) := \int_0^{\alpha} y_1(\theta,\alpha')\cdot r_s(\alpha')\,d\alpha' \qquad \eta_{1s}(\theta) := \frac{1}{L}\cdot s\eta_1(\theta,\psi_0) \qquad \eta_{1s}(\theta_0) = 0.037\,mm$$

$$sp2_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')^2\cdot r_s(\alpha')\,d\alpha' \qquad p2_1(\theta) := \frac{1}{L}\cdot sp2_1(\theta,\psi_0) \qquad p2_1(\theta_0) = 1.207\,mm^2$$

$$sq2_1(\theta,\alpha) := \int_0^{\alpha} y_1(\theta,\alpha')^2\cdot r_s(\alpha')\,d\alpha' \qquad q2_1(\theta) := \frac{1}{L}\cdot sq2_1(\theta,\psi_0) \qquad q2_1(\theta_0) = 1.168\,mm^2$$

$$sk_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')\cdot y_1(\theta,\alpha')\cdot r_s(\alpha')\,d\alpha' \qquad k_1(\theta) := \frac{1}{L}\cdot sk_1(\theta,\psi_0) \qquad k_1(\theta_0) = 0.027\,mm^2$$

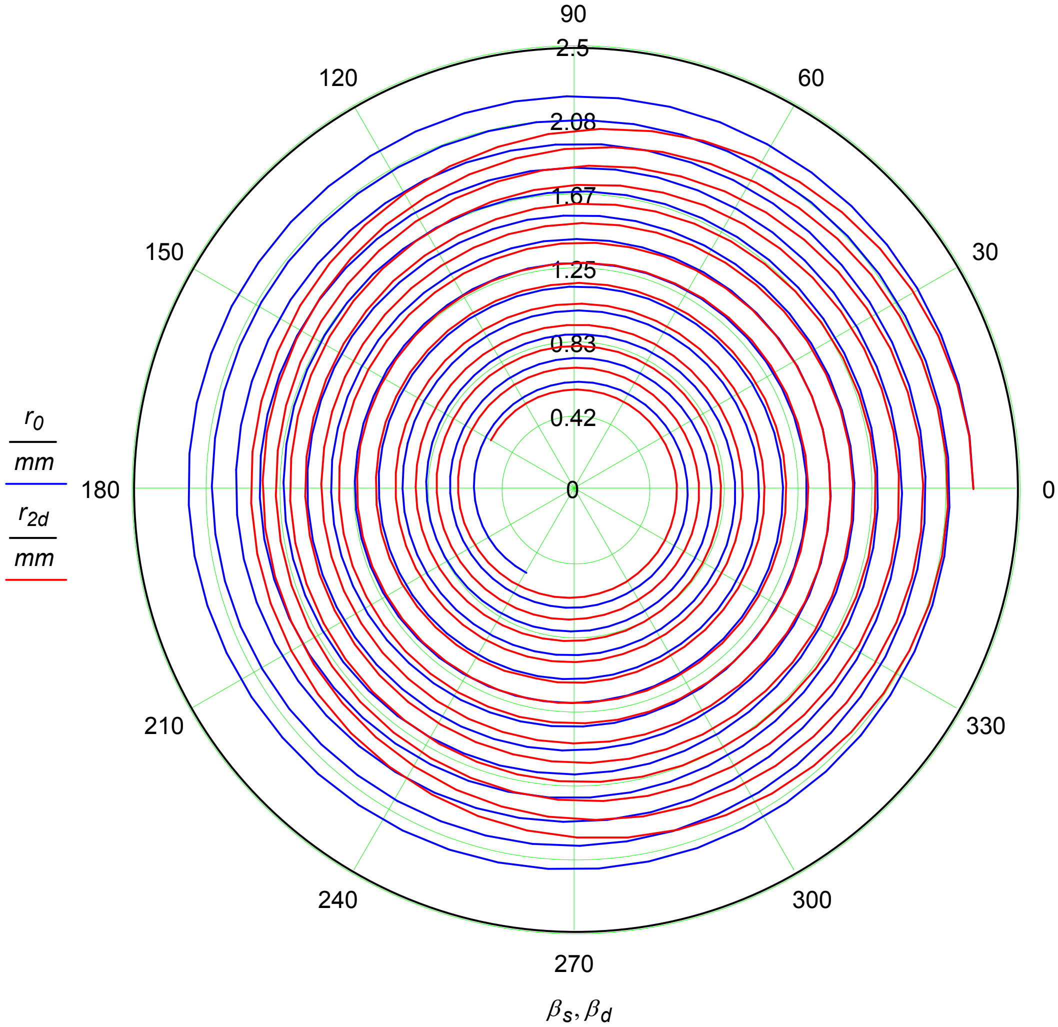
$$\mathbf{S_1}(\theta,\alpha) := \frac{1}{E\cdot I_{33}}\cdot\begin{pmatrix} -y_1(\theta,\alpha)\cdot s\eta_1(\theta,\alpha) + sq2_1(\theta,\alpha) & y_1(\theta,\alpha)\cdot s\xi_1(\theta,\alpha) - sk_1(\theta,\alpha) \\ x_1(\theta,\alpha)\cdot s\eta_1(\theta,\alpha) - sk_1(\theta,\alpha) & -x_1(\theta,\alpha)\cdot s\xi_1(\theta,\alpha) + sp2_1(\theta,\alpha) \end{pmatrix}$$

$$\mathbf{R'}(\theta) := \mathbf{S_1}(\theta,\psi_0)^{-1}\cdot\begin{pmatrix} x_V(\theta) - x_1(\theta,\psi_0) \\ y_V(\theta) - y_1(\theta,\psi_0) \end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} -1.128\times10^{-4} \\ 1.872\times10^{-6} \end{pmatrix} N \qquad \mathbf{\Delta z_s}(\theta,\alpha) := \mathbf{S_1}(\theta,\alpha)\cdot\mathbf{R'}(\theta)$$

$$\Delta z_1(\theta,\alpha) := \mathbf{\Delta z_s}(\theta,\alpha)_0 + i\cdot\mathbf{\Delta z_s}(\theta,\alpha)_1 \qquad z_2(\theta,\alpha) := z_1(\theta,\alpha) + \Delta z_1(\theta,\alpha)$$

$$z_{2d_i} := z_2(\theta_0,\alpha_i) \qquad n_{pt} := dernier(z_d) \qquad x_{2d} := Re(z_{2d}) \qquad y_{2d} := Im(z_{2d}) \qquad r_{2d} := \overrightarrow{|z_{2d}|} \qquad r_{2d_{npt}} = 0.55\,mm$$

$$\beta_d := \overrightarrow{Atan(x_{2d}, y_{2d})} \qquad \beta_{d_0} = 0\,deg \qquad \beta_{d_{npt}} = 150\,deg \qquad mod(\alpha_V(\theta_0), 2\cdot\pi) = 150\,deg$$



$$x_V(\theta_0) = -0.476\,mm \qquad x_{2d_{n-1}} - x_V(\theta_0) = 0\,mm \qquad y_V(\theta_0) = 0.275\,mm \qquad y_{2d_{n-1}} - y_V(\theta_0) = 0\,mm$$